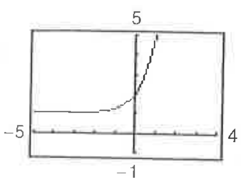
3 Review Exercises

3.1 In Exercises 1–4, use a calculator to evaluate the function at the indicated value of x. Round your result to four decimal places.

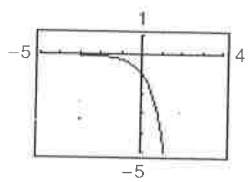
Function	*Value*
1. $f(x) = 1.45^x$	$x = 2\pi$
2. $f(x) = 7^x$	$x = -\sqrt{11}$
3. $g(x) = 60^{2x}$	$x = -1.1$
4. $g(x) = 25^{-3x}$	$x = \frac{3}{2}$

In Exercises 5–8, match the function with its graph. [The graphs are labeled (a), (b), (c), and (d).]

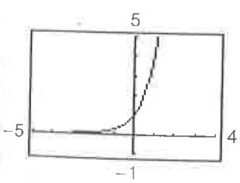
(a)



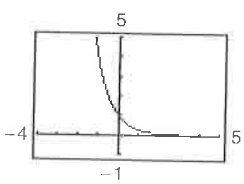
(b)



(c)



(d)



5. $f(x) = 4^x$
6. $f(x) = 4^{-x}$
7. $f(x) = -4^x$
8. $f(x) = 4^x + 1$

In Exercises 9–12, graph the exponential function by hand. Identify any asymptotes and intercepts and determine whether the graph of the function is increasing or decreasing.

9. $f(x) = 6^x$
10. $f(x) = 0.3^{x+1}$
11. $g(x) = 1 + 6^{-x}$
12. $g(x) = 0.3^{-x}$

In Exercises 13–16, use a calculator to evaluate the function $f(x) = e^x$ for the indicated value of x. Round your result to three decimal places.

13. $x = 8$
14. $x = \sqrt{5}$
15. $x = -2.1$
16. $x = -\frac{3}{5}$

In Exercises 17–22, use a graphing utility to construct a table of values for the function. Then sketch the graph of the function.

17. $h(x) = e^{x-1}$
18. $f(x) = e^{x+2}$
19. $h(x) = -e^x$
20. $f(x) = 3 - e^{-x}$
21. $f(x) = 4e^{-0.5x}$
22. $f(x) = 2 + e^{x+3}$

In Exercises 23–28, use a graphing utility to graph the exponential function. Identify any asymptotes of the graph.

23. $g(t) = 8 - 0.5e^{-t/4}$
24. $h(x) = 12(1 + e^{-x/2})$
25. $g(x) = 200e^{4/x}$
26. $f(x) = -8e^{-4/x}$
27. $f(x) = \dfrac{10}{1 + 2^{-0.05x}}$
28. $f(x) = -\dfrac{12}{1 + 4^{-x}}$

Compound Interest **In Exercises 29 and 30, complete the table to determine the balance A for \$10,000 invested at rate r for t years, compounded continuously.**

t	1	10	20	30	40	50
A						

29. $r = 8\%$
30. $r = 3\%$

31. ***Depreciation*** After t years, the value of a car that costs \$26,000 is modeled by

$$V(t) = 26{,}000\left(\frac{3}{4}\right)^t.$$

(a) Use a graphing utility to graph the function.

(b) Find the value of the car 2 years after it was purchased.

(c) According to the model, when does the car depreciate most rapidly? Is this realistic? Explain.

32. ***Radioactive Decay*** Let Q represent a mass of plutonium 241 (^{241}Pu), in grams whose half-life is 14 years. The quantity of plutonium present after t years is given by

$$Q = 100\left(\tfrac{1}{2}\right)^{t/14}.$$

(a) Determine the initial quantity (when $t = 0$).

(b) Determine the quantity present after 10 years.

(c) Use a graphing utility to graph the function over the interval $t = 0$ to $t = 100$.

3.2 In Exercises 33–36, write the exponential equation in logarithmic form.

33. $4^3 = 64$

34. $3^5 = 243$

35. $25^{3/2} = 125$

36. $12^{-1} = \frac{1}{12}$

In Exercises 37–40, evaluate the function at the indicated value of x without using a calculator.

Function	*Value*
37. $f(x) = \log_6 x$	$x = 216$
38. $f(x) = \log_7 x$	$x = 1$
39. $f(x) = \log_4 x$	$x = \frac{1}{4}$
40. $f(x) = \log_{10} x$	$x = 0.001$

In Exercises 41–44, find the domain, vertical asymptote, and x-intercept of the logarithmic function, and sketch its graph by hand. Verify using a graphing utility.

41. $g(x) = -\log_2 x + 5$

42. $g(x) = \log_5(x - 3)$

43. $f(x) = \log_2(x - 1) + 6$

44. $f(x) = \log_5(x + 2) - 3$

In Exercises 45–50, use a calculator to evaluate the function $f(x) = \ln x$ at the indicated value of x. Round your result to three decimal places, if necessary.

45. $x = 21.5$

46. $x = 0.98$

47. $x = e^7$

48. $x = e^{-1/2}$

49. $x = \sqrt{6}$

50. $x = \frac{2}{5}$

In Exercises 51–54, use a graphing utility to graph the logarithmic function. Determine the domain and identify any vertical asymptote and x-intercept.

51. $f(x) = \ln x + 3$

52. $f(x) = \ln(x - 3)$

53. $h(x) = \frac{1}{2} \ln x$

54. $f(x) = \frac{1}{4} \ln x$

55. ***Climb Rate*** The time t (in minutes) for a small plane to climb to an altitude of h feet is given by

$$t = 50 \log_{10} \frac{18{,}000}{18{,}000 - h}$$

where 18,000 feet is the plane's absolute ceiling.

(a) Determine the domain of the function appropriate for the context of the problem.

(b) Use a graphing utility to graph the function and identify any asymptotes.

(c) As the plane approaches its absolute ceiling, what can be said about the time required to further increase its altitude?

(d) Find the amount of time it will take for the plane to climb to an altitude of 4000 feet.

56. ***Home Mortgage*** The model

$$t = 12.542 \ln\left(\frac{x}{x - 1000}\right), \quad x > 1000$$

approximates the length of a home mortgage of \$150,000 at 8% in terms of the monthly payment. In the model, t is the length of the mortgage in years and x is the monthly payment in dollars.

(a) Use the model to approximate the length of a \$150,000 mortgage at 8% when the monthly payment is \$1254.68.

(b) Approximate the total amount paid over the term of the mortgage with a monthly payment of \$1254.68. What amount of the total is interest costs?

3.3 In Exercises 57–60, evaluate the logarithm using the change-of-base formula. Do each problem twice, once with common logarithms and once with natural logarithms. Round your results to three decimal places.

57. $\log_4 9$

58. $\log_{1/2} 5$

59. $\log_{12} 200$

60. $\log_3 0.28$

In Exercises 61–64, use the properties of logarithms to rewrite and simplify the logarithmic expression.

61. $\ln 20$

62. $\ln(3e^{-4})$

63. $\log_5\left(\frac{1}{15}\right)$

64. $\log_{10} \frac{9}{300}$

In Exercises 65–70, use the properties of logarithms to expand the expression as a sum, difference, and/or constant multiple of logarithms. (Assume all variables are positive.)

65. $\log_5 5x^2$

66. $\log_4 3xy^2$

67. $\log_{10} \frac{5\sqrt{y}}{x^2}$

68. $\ln \frac{\sqrt{x}}{4}$

69. $\ln\left(\frac{x + 3}{xy}\right)$

70. $\ln \frac{xy^5}{\sqrt{z}}$

In Exercises 71–76, condense the expression to the logarithm of a single quantity.

71. $\log_2 5 + \log_2 x$
72. $\log_6 y - 2\log_6 z$
73. $\frac{1}{2}\ln(2x - 1) - 2\ln(x + 1)$
74. $5\ln(x - 2) - \ln(x + 2) - 3\ln(x)$
75. $\ln 3 + \frac{1}{3}\ln(4 - x^2) - \ln x$
76. $3[\ln x - 2\ln(x^2 + 1)] + 2\ln 5$

77. ***Snow Removal*** The number of miles s of roads cleared of snow is approximated by the model

$$s = 25 - \frac{13\ln(h/12)}{\ln 3}, \quad 2 \le h \le 15$$

where h is the depth of the snow (in inches).

(a) Use a graphing utility to graph the function.

(b) Complete the table.

h	4	6	8	10	12	14
s						

(c) Using the graph of the function and the table, what conclusion can you make about the miles of roads cleared as the depth of the snow increases?

78. ***Human Memory Model*** Students in a sociology class were given an exam and then retested monthly with an equivalent exam. The average scores for the class are given by the human memory model $f(t) = 85 - 14\log_{10}(t + 1)$, where t is the time in months and $0 \le t \le 10$. When will the average score decrease to 71?

3.4 In Exercises 79–86, solve for x.

79. $8^x = 512$
80. $3^x = 729$
81. $6^x = \frac{1}{216}$
82. $6^{x-2} = 1296$
83. $\log_7 x = 4$
84. $\log_x 243 = 5$
85. $\ln x = 4$
86. $\ln x = -3$

In Exercises 87–96, solve the exponential equation algebraically. Round your result to three decimal places.

87. $e^x = 12$
88. $e^{3x} = 25$
89. $3e^{-5x} = 132$
90. $14e^{3x+2} = 560$
91. $2^x + 13 = 35$
92. $6^x - 28 = -8$
93. $-4(5^x) = -68$
94. $2(12^x) = 190$
95. $e^{2x} - 7e^x + 10 = 0$
96. $e^{2x} - 6e^x + 8 = 0$

In Exercises 97–108, solve the logarithmic equation algebraically. Round your result to three decimal places.

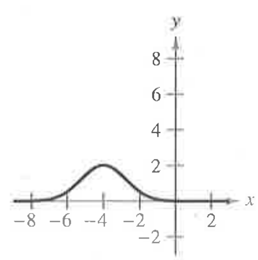
97. $\ln 3x = 8.2$
98. $\ln 5x = 7.2$
99. $2\ln 4x = 15$
100. $4\ln 3x = 15$
101. $\ln x - \ln 3 = 2$
102. $\ln\sqrt{x + 8} = 3$
103. $\ln\sqrt{x + 1} = 2$
104. $\ln x - \ln 5 = 4$
105. $\log_{10}(x - 1) = \log_{10}(x - 2) - \log_{10}(x + 2)$
106. $\log_{10}(x + 2) - \log_{10} x = \log_{10}(x + 5)$
107. $\log_{10}(1 - x) = -1$
108. $\log_{10}(-x - 4) = 2$

109. ***Compound Interest*** You deposit \$7550 into an account that pays 7.25% interest, compounded continuously. How long will it take for the money to triple?

110. ***Demand*** The demand equation for a 32-inch television is modeled by $p = 500 - 0.5e^{0.004x}$. Find the demand x for a price of (a) $p = \$450$ and (b) $p = \$400$.

3.5 In Exercises 111–116, match the function with its graph. [The graphs are labeled (a), (b), (c), (d), (e), and (f).]

(a)



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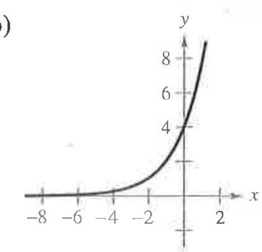

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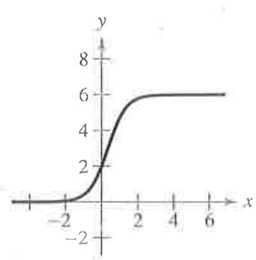

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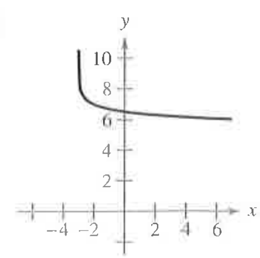

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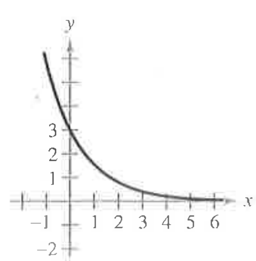
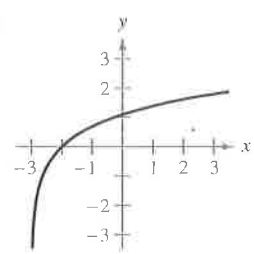


(f)



111. $y = 3e^{-2x/3}$

112. $y = 4e^{2x/3}$

113. $y = \ln(x + 3)$

114. $y = 7 - \log_{10}(x + 3)$

115. $y = 2e^{-(x+4)^2/3}$

116. $y = \dfrac{6}{1 + 2e^{-2x}}$

In Exercises 117–120, find the exponential model $y = ae^{bx}$ that fits the two points.

117. $(0, 2), (4, 3)$

118. $(0, 2), (5, 1)$

119. $\left(0, \frac{1}{2}\right), (5, 5)$

120. $(0, 4), \left(5, \frac{1}{2}\right)$

121. ***Population*** The population P (in thousands) of Colorado Springs, Colorado is given by

$$P = 361e^{kt}$$

where $t = 0$ represents the year 2000. In 1980, the population was 215,000. Find the value of k and use this result to predict the population in the year 2020. (Source: U.S. Census Bureau)

122. ***Radioactive Decay*** The half-life of radioactive uranium II (^{234}U) is 245,500 years. What percent of the present amount of radioactive uranium II will remain after 5000 years?

123. ***Compound Interest*** A deposit of $10,000 is made in a savings account for which the interest is compounded continuously. The balance will double in 12 years.

(a) What is the annual interest rate for this account?

(b) Find the balance after 1 year.

124. ***Test Scores*** The test scores for a biology test follow a normal distribution modeled by

$$y = 0.0499e^{-(x-71)^2/128}$$

where x is the test score.

(a) Use a graphing utility to graph the function.

(b) From the graph in part (a), estimate the average test score.

125. ***Typing Speed*** In a typing class, the average number of words per minute N typed after t weeks of lessons was found to be modeled by

$$N = \frac{157}{1 + 5.4e^{-0.12t}}.$$

Find the number of weeks necessary to type (a) 50 words per minute and (b) 75 words per minute.

126. ***Geology*** On the Richter scale, the magnitude R of an earthquake of intensity I is modeled by

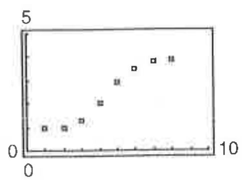
$$R = \log_{10} \frac{I}{I_0}$$

where $I_0 = 1$ is the minimum intensity used for comparison. Find the intensities I of the following earthquakes measuring R on the Richter scale.

(a) $R = 8.4$ (b) $R = 6.85$ (c) $R = 9.1$

3.6 In Exercises 127–130, determine whether the scatter plot could best be modeled by a linear model, a quadratic model, an exponential model, a logarithmic model, or a logistic model.

127.



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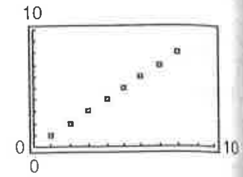

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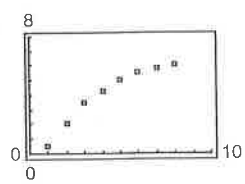
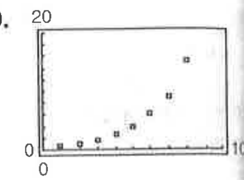


130.



131. ***Entertainment*** The table shows the number M (in thousands) of movie theater screens in the United States for selected years from 1975 to 2000. (Source: Motion Picture Association of America)

Year	Number of screens, M
1975	11
1980	14
1985	18
1990	23
1995	27
2000	37

(a) Use the *regression* feature of a graphing utility to find a quadratic model, an exponential model, and a power model for the data. Let x represent the year, with $x = 5$ corresponding to 1975.

4 Review Exercises

4.1 In Exercises 1 and 2, estimate the angle to the nearest one-half radian.

1. 

2. 

In Exercises 3–6, (a) sketch the angle in standard position, (b) determine the quadrant in which the angle lies, and (c) list one positive and one negative coterminal angle.

3. $\frac{\pi}{16}$
4. $\frac{40\pi}{47}$
5. $-\frac{9\pi}{15}$
6. $-\frac{11\pi}{3}$

In Exercises 7–10, find (if possible) the complement and supplement of the angle.

7. $\frac{\pi}{8}$
8. $\frac{13\pi}{18}$
9. $\frac{3\pi}{10}$
10. $\frac{2\pi}{21}$

In Exercises 11–14, (a) sketch the angle in standard position, (b) determine the quadrant in which the angle lies, and (c) list one positive and one negative coterminal angle.

11. $35°$
12. $190°$
13. $-110°$
14. $-420°$

In Exercises 15–18, find (if possible) the complement and supplement of each angle.

15. $8°$
16. $94°$
17. $171°$
18. $49°$

In Exercises 19–22, use the angle conversion capabilities of a graphing utility to convert the angle measure to decimal degree form. Round your answer to three decimal places.

19. $135° 16' 45''$
20. $-234° 40''$
21. $5° 22' 53''$
22. $280° 8' 50''$

In Exercises 23–26, use the angle-conversion capabilities of a graphing utility to convert the angle measure to D°M′S″ form.

23. $135.29°$
24. $25.8°$
25. $-85.36°$
26. $-327.93°$

In Exercises 27–30, convert the angle measure from degrees to radians. Round your answer to four decimal places.

27. $480°$
28. $-16.5°$
29. $-33°$
30. $84°$

In Exercises 31–34, convert the angle measure from radians to degrees. Round your answer to three decimal places.

31. $\frac{5\pi}{7}$
32. $-\frac{3\pi}{5}$
33. -3.5
34. 1.55

35. Find the radian measure of the central angle of a circle with a radius of 12 feet that intercepts an arc of length 25 feet.

36. Find the radian measure of the central angle of a circle with a radius of 60 inches that intercepts an arc of length 245 inches.

37. Find the length of the arc on a circle with a radius of 20 meters intercepted by a central angle of 138°.

38. Find the length of the arc on a circle with a radius of 15 centimeters intercepted by a central angle of 60°.

39. *Music* The radius of a compact disc is 6 centimeters. Find the linear speed of a point on the circumference of the disc if it is rotating at a speed of 500 revolutions per minute.

40. *Angular Speed* A car is moving at a rate of 28 miles per hour, and the diameter of its wheels is about $2\frac{1}{3}$ feet.

    (a) Find the number of revolutions per minute the wheels are rotating.

    (b) Find the angular speed of the wheels in radians per minute.

4.2 In Exercises 41–44, find the point (x, y) on the unit circle that corresponds to the real number t.

41. $t = \dfrac{7\pi}{4}$

42. $t = \dfrac{3\pi}{4}$

43. $t = \dfrac{5\pi}{6}$

44. $t = -\dfrac{4\pi}{3}$

In Exercises 45–48, evaluate (if possible) the six trigonometric functions of the real number.

45. $t = \dfrac{7\pi}{6}$

46. $t = \dfrac{\pi}{4}$

47. $t = -\dfrac{2\pi}{3}$

48. $t = \pi$

In Exercises 49–52, evaluate the trigonometric function using its period as an aid.

49. $\sin \dfrac{11\pi}{4}$

50. $\cos 4\pi$

51. $\sin\left(-\dfrac{17\pi}{6}\right)$

52. $\cos\left(-\dfrac{13\pi}{3}\right)$

In Exercises 53–56, use a calculator to evaluate the expression. Round your answer to four decimal places.

53. $\cot 2.3$

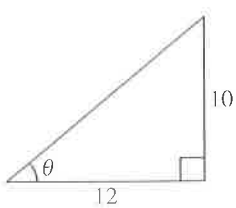
54. $\sec 4.5$

55. $\cos \dfrac{5\pi}{3}$

56. $\tan\left(-\dfrac{11\pi}{6}\right)$

4.3 In Exercises 57–60, find the exact values of the six trigonometric functions of the angle θ shown in the figure.

57.



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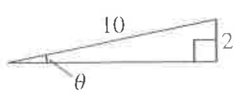

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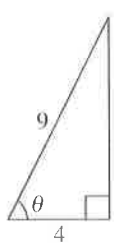

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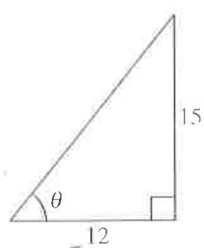


In Exercises 61 and 62, use trigonometric identities to transform one side of the equation into the other.

61. $\csc \theta \tan \theta = \sec \theta$

62. $\dfrac{\cot \theta + \tan \theta}{\cot \theta} = \sec^2 \theta$

In Exercises 63–66, use a calculator to evaluate each function. Round your answers to four decimal places.

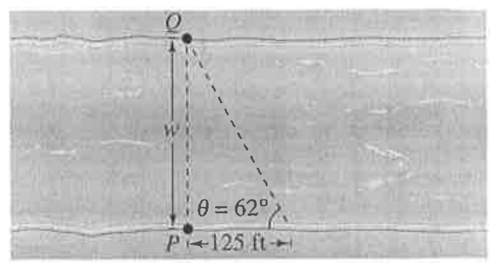
63. (a) $\cos 84°$ (b) $\sin 6°$

64. (a) $\csc 52° 12'$ (b) $\sec 54° 7'$

65. (a) $\cos \dfrac{\pi}{4}$ (b) $\sec \dfrac{\pi}{4}$

66. (a) $\tan \dfrac{3\pi}{20}$ (b) $\cot \dfrac{3\pi}{20}$

67. ***Width*** An engineer is trying to determine the width of a river. From point P, the engineer walks downstream 125 feet and sights to point Q. From this sighting, it is determined that $\theta = 62°$. How wide is the river?



68. ***Height*** An escalator 152 feet in length rises to a platform and makes a 30° angle with the ground.

(a) Draw a right triangle that gives a visual representation of the problem. Show the known quantities of the triangle and use a variable to indicate the height of the platform above the ground.

(b) Use a trigonometric function to write an equation involving the unknown quantity.

(c) Find the height of the platform above the ground.

4.4 In Exercises 69–74, the point is on the terminal side of an angle in standard position. Determine the exact values of the six trigonometric functions of the angle.

69. $(12, 16)$

70. $(-4, -6)$

71. $(-7, 2)$

72. $(4, -8)$

73. $(2, 5)$

74. $(-9, 3)$

In Exercises 75–78, find the values of the six trigonometric functions of θ satisfying the given conditions.

75. $\sec \theta = \frac{6}{5}, \quad \tan \theta < 0$

76. $\tan \theta = -\frac{12}{5}, \quad \sin \theta > 0$

77. $\sin \theta = \frac{3}{8}, \quad \cos \theta < 0$

78. $\cos \theta = -\frac{2}{5}, \quad \sin \theta > 0$

In Exercises 79–82, find the reference angle θ' and sketch θ and θ' in standard position.

79. $\theta = 264°$ **80.** $\theta = 635°$

81. $\theta = -\dfrac{6\pi}{5}$ **82.** $\theta = \dfrac{17\pi}{3}$

In Exercises 83–90, evaluate the sine, cosine, and tangent of the angle without using a calculator.

83. $240°$ **84.** $315°$

85. $-210°$ **86.** $-315°$

87. $-\dfrac{9\pi}{4}$ **88.** $\dfrac{11\pi}{6}$

89. $\dfrac{\pi}{2}$ **90.** $-\dfrac{\pi}{3}$

In Exercises 91–94, use a calculator to evaluate the trigonometric function. Round to four decimal places.

91. $\tan 33°$ **92.** $\csc 105°$

93. $\sec \dfrac{12\pi}{5}$ **94.** $\sin\left(-\dfrac{\pi}{9}\right)$

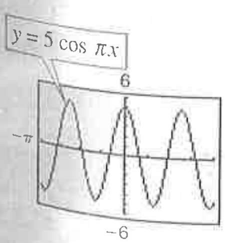
4.5 In Exercises 95–98, sketch the graph of the function.

95. $f(x) = 3 \sin x$ **96.** $f(x) = 2 \cos x$

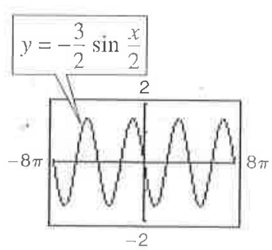
97. $f(x) = \frac{1}{4} \cos x$ **98.** $f(x) = \frac{7}{2} \sin x$

In Exercises 99–102, find the period and amplitude.

99.



100.



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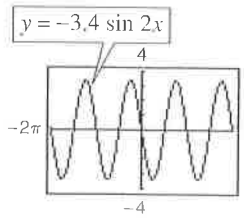

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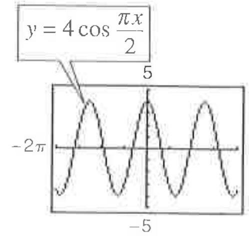


In Exercises 103–114, sketch the graph of the function. (Include two full periods.)

103. $f(x) = 3 \cos 2\pi x$ **104.** $f(x) = -2 \sin \pi x$

105. $f(x) = 5 \sin \dfrac{2x}{5}$ **106.** $f(x) = 8 \cos\left(-\dfrac{x}{4}\right)$

107. $f(x) = -\dfrac{5}{2} \cos \dfrac{x}{4}$ **108.** $f(x) = -\dfrac{1}{2} \sin \dfrac{\pi x}{4}$

109. $f(x) = \frac{5}{2} \sin(x - \pi)$ **110.** $f(x) = 3 \cos(x + \pi)$

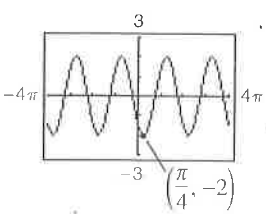
111. $f(x) = 2 - \cos \dfrac{\pi x}{2}$ **112.** $f(x) = \frac{1}{2} \sin \pi x - 3$

113. $f(x) = -3 \cos\left(\dfrac{x}{2} - \dfrac{\pi}{4}\right)$

114. $f(x) = 4 - 2 \cos(4x + \pi)$

Graphical Reasoning **In Exercises 115–118, find a, b, and c for the function $f(x) = a \cos(bx - c)$ such that the graph of f matches the graph shown.**

115.



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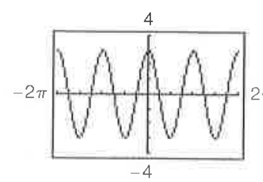

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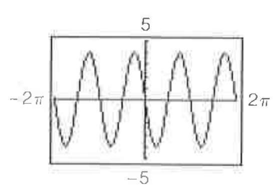

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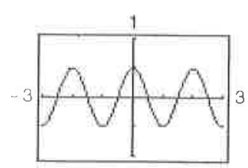


Sales **In Exercises 119 and 120, use a graphing utility to graph the sales function over 1 year, where S is the sales (in thousands of units) and t is the time (in months), with $t = 1$ corresponding to January. Determine the months of maximum and minimum sales.**

119. $S = 48.4 - 6.1 \cos \frac{\pi t}{6}$

120. $S = 56.25 + 9.50 \sin \frac{\pi t}{6}$

4.6 In Exercises 121–140, sketch the graph of the function. (Include two full periods.)

121. $f(x) = -\tan \frac{\pi x}{4}$

122. $f(x) = 4 \tan \pi x$

123. $f(x) = \frac{1}{4} \tan \frac{\pi x}{2}$

124. $f(x) = \tan\left(x + \frac{\pi}{4}\right)$

125. $f(x) = -\frac{1}{4} \tan\left(x - \frac{\pi}{2}\right)$

126. $f(x) = 2 + 2 \tan \frac{x}{3}$

127. $f(x) = 3 \cot \frac{x}{2}$

128. $f(x) = \frac{1}{2} \cot \frac{\pi x}{2}$

129. $f(x) = \frac{1}{2} \cot\left(x - \frac{\pi}{2}\right)$

130. $f(x) = 4 \cot\left(x + \frac{\pi}{4}\right)$

131. $f(x) = \frac{1}{4} \sec x$

132. $f(x) = \frac{1}{2} \csc x$

133. $f(x) = \frac{1}{4} \csc 2x$

134. $f(x) = \frac{1}{2} \sec 2\pi x$

135. $f(x) = \sec\left(x - \frac{\pi}{4}\right)$

136. $f(x) = \frac{1}{2} \csc(2x + \pi)$

137. $f(x) = 2 \sec(x - \pi)$

138. $f(x) = -2 \csc(x - \pi)$

139. $f(x) = \csc\left(3x - \frac{\pi}{2}\right)$

140. $f(x) = 3 \csc\left(2x + \frac{\pi}{4}\right)$

In Exercises 141–144, use a graphing utility to graph the function and the damping factor of the function in the same viewing window. Describe the behavior of the function as x increases without bound.

141. $f(x) = e^x \sin 2x$

142. $f(x) = 2x \cos x$

143. $f(x) = e^x \cos x$

144. $f(x) = x \sin \pi x$

4.7 In Exercises 145–148, find the value of each expression without using a calculator.

145. (a) $\arcsin 1$ (b) $\arcsin 4$

146. (a) $\arcsin\left(-\frac{\sqrt{2}}{2}\right)$ (b) $\arcsin \frac{\sqrt{3}}{2}$

147. (a) $\cos^{-1} \frac{\sqrt{2}}{2}$ (b) $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

148. (a) $\tan^{-1}(-\sqrt{3})$ (b) $\tan^{-1} 1$

In Exercises 149–156, use a calculator to approximate the value of the expression. Round your answer to two decimal places.

149. $\arccos 0.42$

150. $\arcsin 0.63$

151. $\sin^{-1}(-0.94)$

152. $\cos^{-1}(-0.12)$

153. $\arctan(-12)$

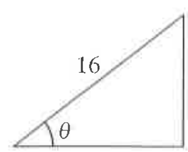
154. $\arctan 21$

155. $\tan^{-1} 0.81$

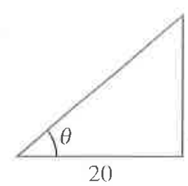
156. $\tan^{-1} 6.4$

In Exercises 157 and 158, use an inverse trigonometric function to write θ as a function of x.

157.



158.



In Exercises 159–162, write an algebraic expression that is equivalent to the expression.

159. $\sec[\arcsin(x - 1)]$

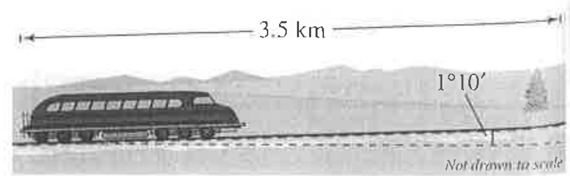
160. $\tan\left(\arccos \frac{x}{2}\right)$

161. $\sin\left(\arccos \frac{x^2}{4 - x^2}\right)$

162. $\csc(\arcsin 10x)$

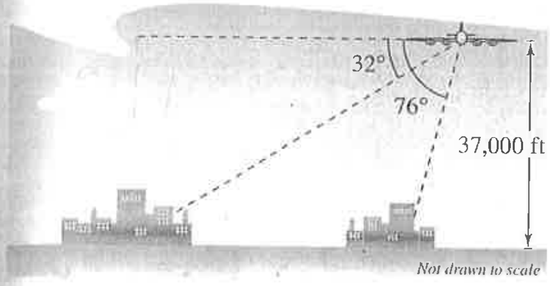
4.8

163. ***Railroad Grade*** A train travels 3.5 kilometers on a straight track with a grade of $1° 10'$. What is the vertical rise of the train in that distance?



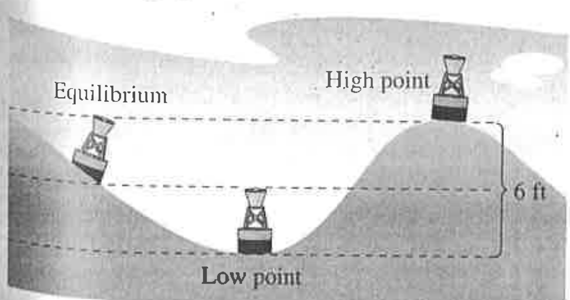
164. ***Mountain Descent*** A road sign at the top of a mountain indicates that for the next 4 miles the grade is 12%. Find the angle of the grade and the change in elevation for a car descending the mountain.

165. ***Distance*** A passenger in an airplane flying at an altitude of 37,000 feet sees two towns directly to the west of the airplane. The angles of depression to the towns are 32° and 76° (see figure). How far apart are the towns?



166. ***Distance*** From city A to city B, a plane flies 650 miles at a bearing of 48°. From city B to city C, the plane flies 810 miles at a bearing of 115°. Find the distance from A to C and the bearing from A to C.

167. ***Wave Motion*** A buoy oscillates in simple harmonic motion as waves go past. At a given time it is noted that the buoy moves a total of 6 feet from its low point to its high point, returning to its high point every 15 seconds (see figure). Write an equation that describes the motion of the buoy if it is at its high point at $t = 0$.



168. ***Wave Motion*** Your fishing bobber oscillates in simple harmonic motion from the waves in the lake where you fish. Your bobber moves a total of 1.5 inches from its high point to its low point and returns to its high point every 3 seconds. Write an equation modeling the motion of your bobber if it is at its high point at $t = 0$.

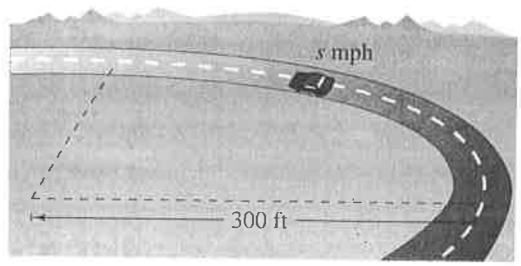
Synthesis

True or False? **In Exercises 169 and 170, determine whether the statement is true or false. Justify your answer.**

169. $y = \sin\theta$ is not a function because $\sin 30° = \sin 150°$.

170. The tangent function is often useful for modeling simple harmonic motion.

171. ***Numerical Analysis*** A 3000-pound automobile is negotiating a circular interchange of radius 300 feet at a speed of s miles per hour (see figure). The relationship between the speed and the angle θ (in degrees) at which the roadway should be banked so that no lateral frictional force is exerted on the tires is $\tan\theta = 0.672s^2/3000$.



(a) Use a graphing utility to complete the table.

s	10	20	30	40	50	60
θ						

(b) In the table, s is incremented by 10, but θ does not increase by equal increments. Explain.

172. ***Approximation*** In calculus it can be shown that the arctangent function can be approximated by the polynomial

$$\arctan x \approx x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7}$$

where x is in radians.

(a) Use a graphing utility to graph the arctangent function and its polynomial approximation in the same viewing window. How do the graphs compare?

(b) Study the pattern in the polynomial approximation of the arctangent function and guess the next term. Then repeat part (a). How does the accuracy of the approximation change when additional terms are added?

5 Review Exercises

5.1 In Exercises 1–10, name the trigonometric function equivalent to the expression.

1. $\dfrac{1}{\cos x}$
2. $\dfrac{1}{\sin x}$
3. $\dfrac{1}{\sec x}$
4. $\dfrac{1}{\tan x}$
5. $\sqrt{1-\cos^2 x}$
6. $\sqrt{1+\tan^2 x}$
7. $\csc\left(\dfrac{\pi}{2}-x\right)$
8. $\cot\left(\dfrac{\pi}{2}-x\right)$
9. $\sec(-x)$
10. $\tan(-x)$

In Exercises 11–14, use the given values to evaluate (if possible) all six trigonometric functions of the angle.

11. $\sin x = \dfrac{4}{5}$, $\quad \cos x = \dfrac{3}{5}$
12. $\tan \theta = \dfrac{2}{3}$, $\quad \sec \theta = \dfrac{\sqrt{13}}{3}$
13. $\sin\left(\dfrac{\pi}{2}-x\right) = \dfrac{1}{\sqrt{2}}$, $\quad \sin x = -\dfrac{1}{\sqrt{2}}$
14. $\csc\left(\dfrac{\pi}{2}-\theta\right) = 3$, $\quad \sin \theta = \dfrac{2\sqrt{2}}{3}$

In Exercises 15–22, use the fundamental identities to simplify the expression. Use the *table* feature of a graphing utility to check your result numerically.

15. $\dfrac{1}{\cot^2 x + 1}$
16. $\dfrac{\sec^2 x - 1}{\sec x - 1}$
17. $\dfrac{\sin^2 \alpha - \cos^2 \alpha}{\sin^2 \alpha - \sin \alpha \cos \alpha}$
18. $\dfrac{\sin^3 \beta + \cos^3 \beta}{\sin \beta + \cos \beta}$
19. $\tan^2 \theta(\csc^2 \theta - 1)$
20. $\csc^2 x(1 - \cos^2 x)$
21. $\tan\left(\dfrac{\pi}{2}-x\right)\sec x$
22. $\dfrac{\sin(-x)\cot x}{\sin\left(\dfrac{\pi}{2}-x\right)}$

23. ***Rate of Change*** The rate of change of the function $f(x) = 2\sqrt{\sin x}$ is given by the expression $\sin^{-1/2} x \cos x$. Show that this expression can also be written as $\cot x\sqrt{\sin x}$.

24. ***Rate of Change*** The rate of change of the function $f(x) = \csc x - \cot x$ is the expression $\csc^2 x - \csc x \cot x$. [Sh]ow that this expression can also be written as $(1 - \cos x)/\sin^2 x$.

5.2 In Exercises 25–36, verify the identity.

25. $\cos x(\tan^2 x + 1) = \sec x$
26. $\sec^2 x \cot x - \cot x = \tan x$
27. $\sin^3 \theta + \sin \theta \cos^2 \theta = \sin \theta$
28. $\cot^2 x - \cos^2 x = \cot^2 x \cos^2 x$
29. $\sin^5 x \cos^2 x = (\cos^2 x - 2\cos^4 x + \cos^6 x)\sin x$
30. $\cos^3 x \sin^2 x = (\sin^2 x - \sin^4 x)\cos x$
31. $\sqrt{\dfrac{1-\sin\theta}{1+\sin\theta}} = \dfrac{1-\sin\theta}{|\cos\theta|}$
32. $\sqrt{1-\cos x} = \dfrac{|\sin x|}{\sqrt{1+\cos x}}$
33. $\dfrac{\csc(-x)}{\sec(-x)} = -\cot x$
34. $\dfrac{1+\sec(-x)}{\sin(-x)+\tan(-x)} = -\csc x$
35. $\sin^2 x + \sin^2\left(\dfrac{\pi}{2}-x\right) = 1$
36. $\csc x \sin\left(\dfrac{\pi}{2}-x\right) = \cot x$

5.3 In Exercises 37–48, solve the equation.

37. $2\sin x - 1 = 0$
38. $\tan x + 1 = 0$
39. $\sin x = \sqrt{3} - \sin x$
40. $4\cos x = 1 + 2\cos x$
41. $3\sqrt{3}\tan x = 3$
42. $\frac{1}{2}\sec x - 1 = 0$
43. $3\csc^2 x = 4$
44. $4\tan^2 x - 1 = \tan^2 x$
45. $4\cos^2 x - 3 = 0$
46. $\sin x(\sin x + 1) = 0$
47. $\sin x - \tan x = 0$
48. $\csc x - 2\cot x = 0$

In Exercises 49–58, find all solutions of the equation in the interval $[0, 2\pi)$. Use a graphing utility to check your answers.

49. $2\cos^2 x - \cos x = 1$
50. $2\sin^2 x - 3\sin x = -1$
51. $\cos^2 x + \sin x = 1$
52. $\sin^2 x + 2\cos x = 2$
53. $2\sin 2x - \sqrt{2} = 0$
54. $\sqrt{3}\tan 3x = 0$
55. $\cos 4x(\cos x - 1) = 0$
56. $3\csc^2 5x = -4$

57. $\cos 4x - 7\cos 2x = 8$

58. $\sin 4x - \sin 2x = 0$

In Exercises 59–62, use the inverse functions where necessary to find all solutions of the equation in the interval $[0, 2\pi)$.

59. $\sin^2 x - 2\sin x = 0$

60. $2\cos^2 x + 3\cos x = 0$

61. $\tan^2 \theta + \tan \theta - 12 = 0$

62. $\sec^2 x + 6\tan x + 4 = 0$

5.4 In Exercises 63–66, find the exact values of the sine, cosine, and tangent of the angle.

63. $285° = 315° - 30°$

64. $345° = 300° + 45°$

65. $\frac{25\pi}{12} = \frac{11\pi}{6} + \frac{\pi}{4}$

66. $\frac{19\pi}{12} = \frac{11\pi}{6} - \frac{\pi}{4}$

In Exercises 67–70, write the expression as the sine, cosine, or tangent of an angle.

67. $\sin 140° \cos 50° + \cos 140° \sin 50°$

68. $\cos 45° \cos 120° - \sin 45° \sin 120°$

69. $\frac{\tan 25° + \tan 10°}{1 - \tan 25° \tan 10°}$

70. $\frac{\tan 68° - \tan 115°}{1 + \tan 68° \tan 115°}$

In Exercises 71–76, find the exact value of the trigonometric function given that $\sin u = \frac{3}{4}$ and $\cos v = -\frac{5}{13}$. (Both u and v are in Quadrant II.)

71. $\sin(u + v)$

72. $\tan(u + v)$

73. $\tan(u - v)$

74. $\sin(u - v)$

75. $\cos(u + v)$

76. $\cos(u - v)$

In Exercises 77–82, verify the identity.

77. $\cos\left(x + \frac{\pi}{2}\right) = -\sin x$

78. $\sin\left(x - \frac{3\pi}{2}\right) = \cos x$

79. $\cot\left(\frac{\pi}{2} - x\right) = \tan x$

80. $\sin(\pi - x) = \sin x$

81. $\cos 3x = 4\cos^3 x - 3\cos x$

82. $\frac{\sin(\alpha + \beta)}{\cos \alpha \cos \beta} = \tan \alpha + \tan \beta$

In Exercises 83 and 84, find the solutions of the equation in the interval $[0, 2\pi)$.

83. $\sin\left(x + \frac{\pi}{2}\right) - \sin\left(x - \frac{\pi}{2}\right) = \sqrt{3}$

84. $\cos\left(x + \frac{3\pi}{4}\right) - \cos\left(x - \frac{3\pi}{4}\right) = 0$

5.5 In Exercises 85–88, find the exact values of $\sin 2u$, $\cos 2u$, and $\tan 2u$ using the double-angle formulas.

85. $\sin u = -\frac{5}{7}, \quad \pi < u < \frac{3\pi}{2}$

86. $\cos u = \frac{4}{5}, \quad \frac{3\pi}{2} < u < 2\pi$

87. $\tan u = -\frac{2}{9}, \quad \frac{\pi}{2} < u < \pi$

88. $\cos u = -\frac{2}{\sqrt{5}}, \quad \frac{\pi}{2} < u < \pi$

In Exercises 89–92, use double-angle formulas to verify the identity algebraically. Use a graphing utility to check your result graphically.

89. $6\sin x \cos x = 3\sin 2x$

90. $4\sin x \cos x + 2 = 2\sin 2x + 2$

91. $1 - 4\sin^2 x \cos^2 x = \cos^2 2x$

92. $\sin 4x = 8\cos^3 x \sin x - 4\cos x \sin x$

93. ***Projectile Motion*** A baseball leaves the hand of the first baseman at an angle of θ with the horizontal and with an initial velocity of $v_0 = 80$ feet per second. The ball is caught by the second baseman 100 feet away. Find θ if the range r of a projectile is given by $r = \frac{1}{32} v_0^2 \sin 2\theta$.

94. ***Projectile Motion*** Use the equation in Exercise 93 to find θ when a golf ball is hit with an initial velocity of $v_0 = 50$ feet per second and lands 77 feet away.

In Exercises 95–98, use the power-reducing formulas to rewrite the expression in terms of the first power of the cosine.

95. $\sin^6 x$

96. $\cos^4 x \sin^4 x$

97. $\cos^4 2x$

98. $\sin^4 2x$

In Exercises 99–102, use the half-angle formulas to determine the exact values of the sine, cosine, and tangent of the angle.

99. $105°$

100. $67° 30'$

101. $\frac{7\pi}{8}$

102. $\frac{11\pi}{12}$

In Exercises 103–106, find the exact values of $\sin(u/2)$, $\cos(u/2)$, and $\tan(u/2)$ using the half-angle formulas.

103. $\sin u = \frac{3}{5}, \quad 0 < u < \frac{\pi}{2}$

104. $\tan u = \frac{5}{8}, \quad \pi < u < \frac{3\pi}{2}$

105. $\cos u = -\frac{2}{7}, \quad \frac{\pi}{2} < u < \pi$

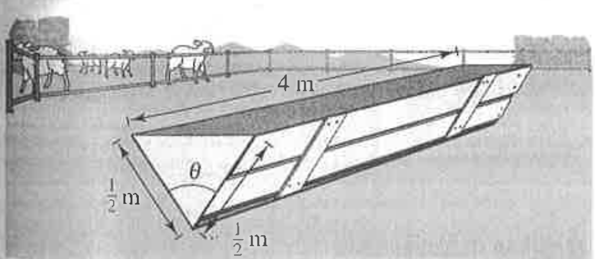
106. $\sec u = -6, \quad \frac{\pi}{2} < u < \pi$

In Exercises 107 and 108, use the half-angle formulas to simplify the expression.

107. $-\sqrt{\frac{1 + \cos 10x}{2}}$

108. $\frac{\sin 6x}{1 + \cos 6x}$

Geometry **In Exercises 109 and 110, a trough for feeding cattle is 4 meters long and its cross sections are isosceles triangles with two equal sides of $\frac{1}{2}$ meter (see figure). The angle between the equal sides is θ.**



109. Write the trough's volume as a function of $\theta/2$.

110. Write the volume of the trough as a function of θ and determine the value of θ such that the volume is maximum.

In Exercises 111–114, use the product-to-sum formulas to write the product as a sum or difference.

111. $6 \sin \frac{\pi}{4} \cos \frac{\pi}{4}$

112. $4 \sin 15° \sin 45°$

113. $\sin 3\alpha \sin 2\alpha$

114. $\cos 4\theta \sin 6\theta$

In Exercises 115–118, use the sum-to-product formulas to write the sum or difference as a product.

115. $\cos 3\theta + \cos 2\theta$

116. $\sin 5\theta + \sin 3\theta$

117. $\sin\left(x + \frac{\pi}{4}\right) - \sin\left(x - \frac{\pi}{4}\right)$

118. $\cos\left(x + \frac{\pi}{6}\right) - \cos\left(x - \frac{\pi}{6}\right)$

Harmonic Motion **In Exercises 119–122, a weight is attached to a spring suspended vertically from a ceiling. When a driving force is applied to the system, the weight moves vertically from its equilibrium position. This motion is described by the model**

$$y = 1.5 \sin 8t - 0.5 \cos 8t$$

where y is the distance from equilibrium in feet and t is the time in seconds.

119. Write the model in the form
$y = \sqrt{a^2 + b^2} \sin(Bt + C)$.

120. Use a graphing utility to graph the model.

121. Find the amplitude of the oscillations of the weight.

122. Find the frequency of the oscillations of the weight.

Synthesis

True or False? **In Exercises 123–126, determine whether the statement is true or false. Justify your answer.**

123. If $\frac{\pi}{2} < \theta < \pi$, then $\cos \frac{\theta}{2} < 0$.

124. $\sin(x + y) = \sin x + \sin y$

125. $4 \sin(-x) \cos(-x) = -2 \sin 2x$

126. $4 \sin 45° \cos 15° = 1 + \sqrt{3}$

127. List the reciprocal identities, quotient identities, and Pythagorean identities from memory.

128. Is $\cos \theta = \sqrt{1 - \sin^2 \theta}$ an identity? Explain.

In Exercises 129 and 130, use the graphs of y_1 and y_2 to determine how to change y_2 to a new function y_3 such that $y_1 = y_3$.

129. $y_1 = \sec^2\left(\frac{\pi}{2} - x\right)$
$y_2 = \cot^2 x$

130. $y_1 = \frac{\cos 3x}{\cos x}$
$y_2 = (2 \sin x)^2$